CSE311 Microwave Engineering

LEC (05)
Transmission Lines_ Part I

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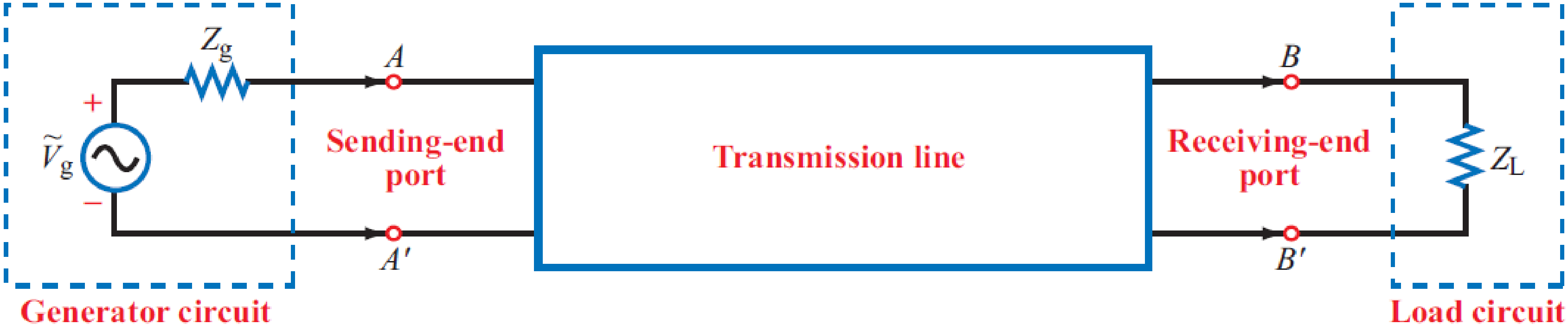
Chapter Contents

3.1 The Lumped–Element Circuit Model for a TL

3.1.1 Transmission line Definition

- Transmission lines are used to transmit electric energy and signals from one point to another, specially from source to load as shown in Fig
- In Radio Frequency (RF) circuits, RF energy has to be transported through:
  - Transmission lines
  - Connectors
- As we transport energy some gets lost due to:
  - Resistance of the wire a $\rightarrow$ lossy cable.
  - Radiation (the energy radiates out of the wire $\rightarrow$ the wire acts as an antenna.

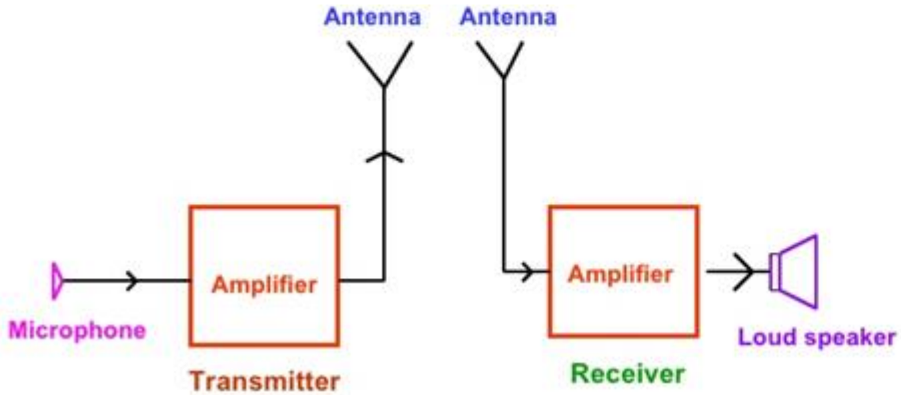


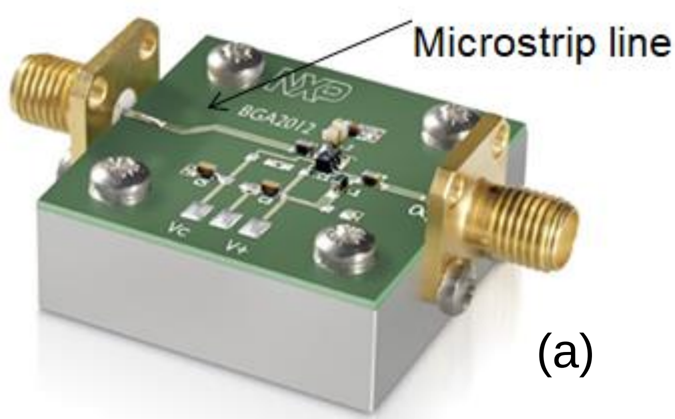
RF transmission line connects a generator to a load.

3.1 The Lumped–Element Circuit Model for a TL

3.1.1 Transmission line Definition

- Transmission lines are commonly met on printed-circuit boards
- The two wires of the transmission line are twisted to reduce interference and radiation from discontinuities.





(a)

A microwave integrated circuit

CAT 5 cable (twisted pair)

(b)

Fig. 3.2 Connection between (a) transmitter and antenna; (b) computers in a network.

3.1 The Lumped–Element Circuit Model for a TL (Continued)

3.1.1 Transmission line Definition

3. Connection between electric generating plant and a substation several hundred miles away Fig. 3.2(d).

Power at low frequency, f = 50 Hz, wavelength, λ is 2×10^6 m

(Over 2000 km)

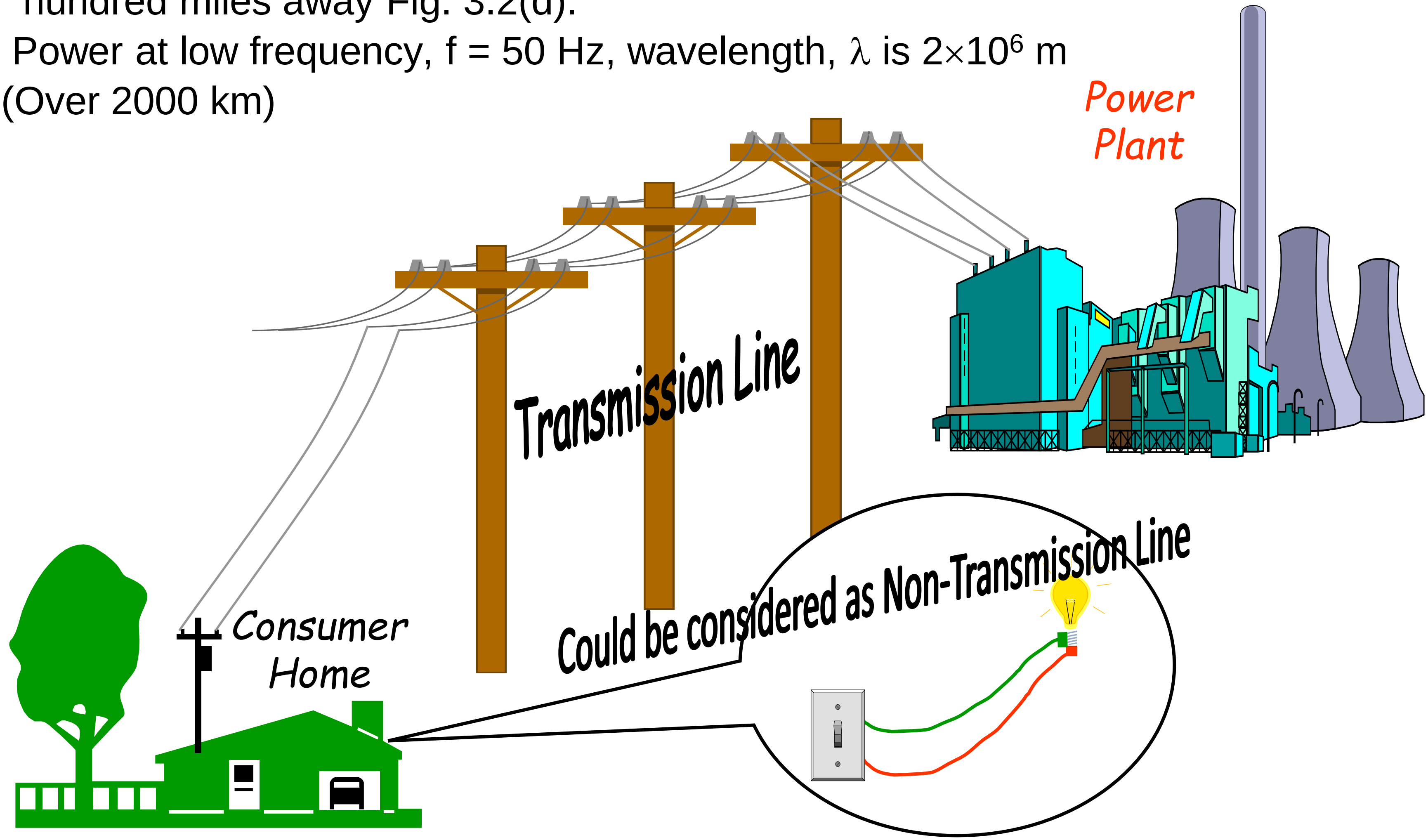


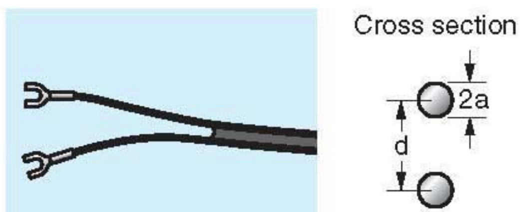
Fig.3.2(c) Power transmission at low frequency power plant (generator) to a load.

3.1 The Lumped–Element Circuit Model for a TL

3.1.1 Transmission line Definition

4. The connection between a cable service provider and your television set.

- TEM (Transverse Electromagnetic): Electric and magnetic fields are orthogonal to one another, and both are orthogonal to direction of propagation



Twin lead

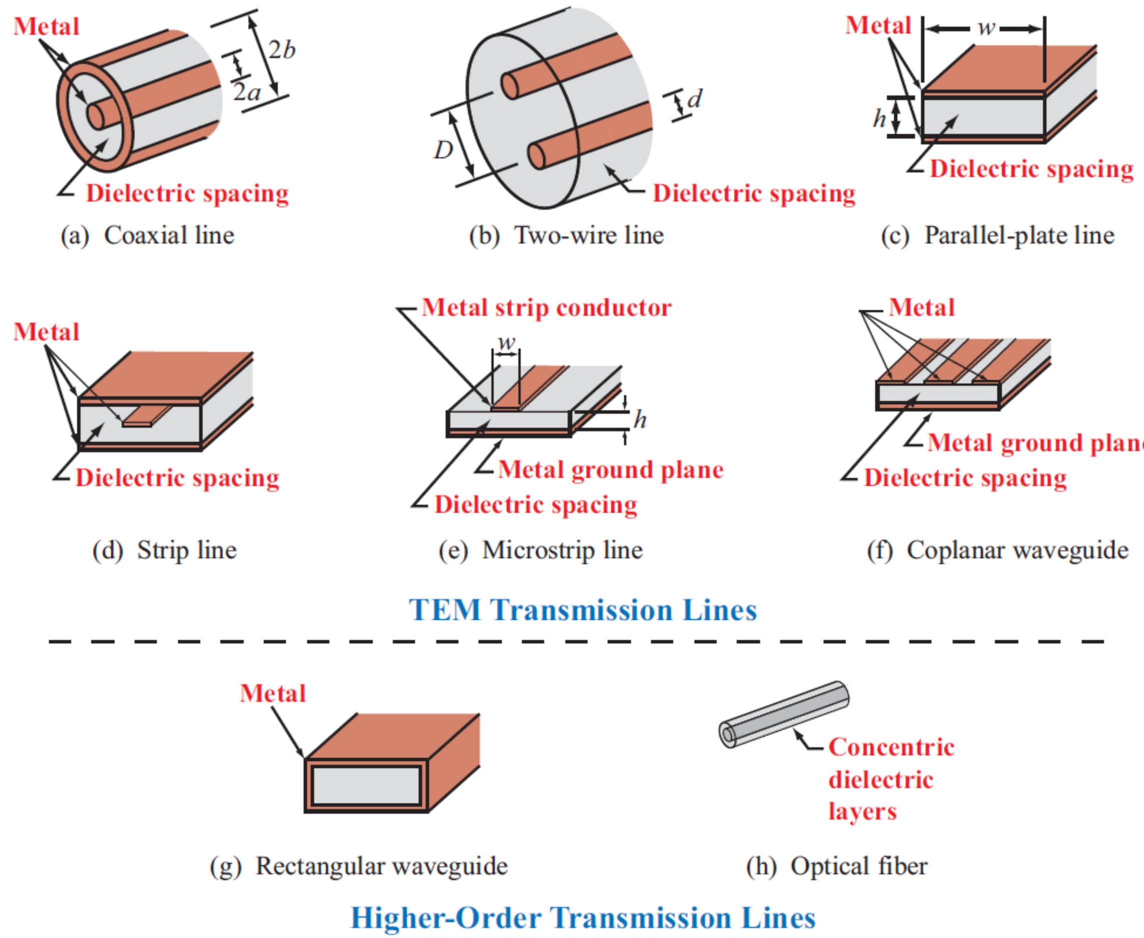


Fig. 3.2(d) Different types of transmission lies and their modes.

3.1 The Lumped–Element Circuit Model for a TL

3.1.1 Transmission line Definition

4. The connection between a cable service provider and your television set are shown:

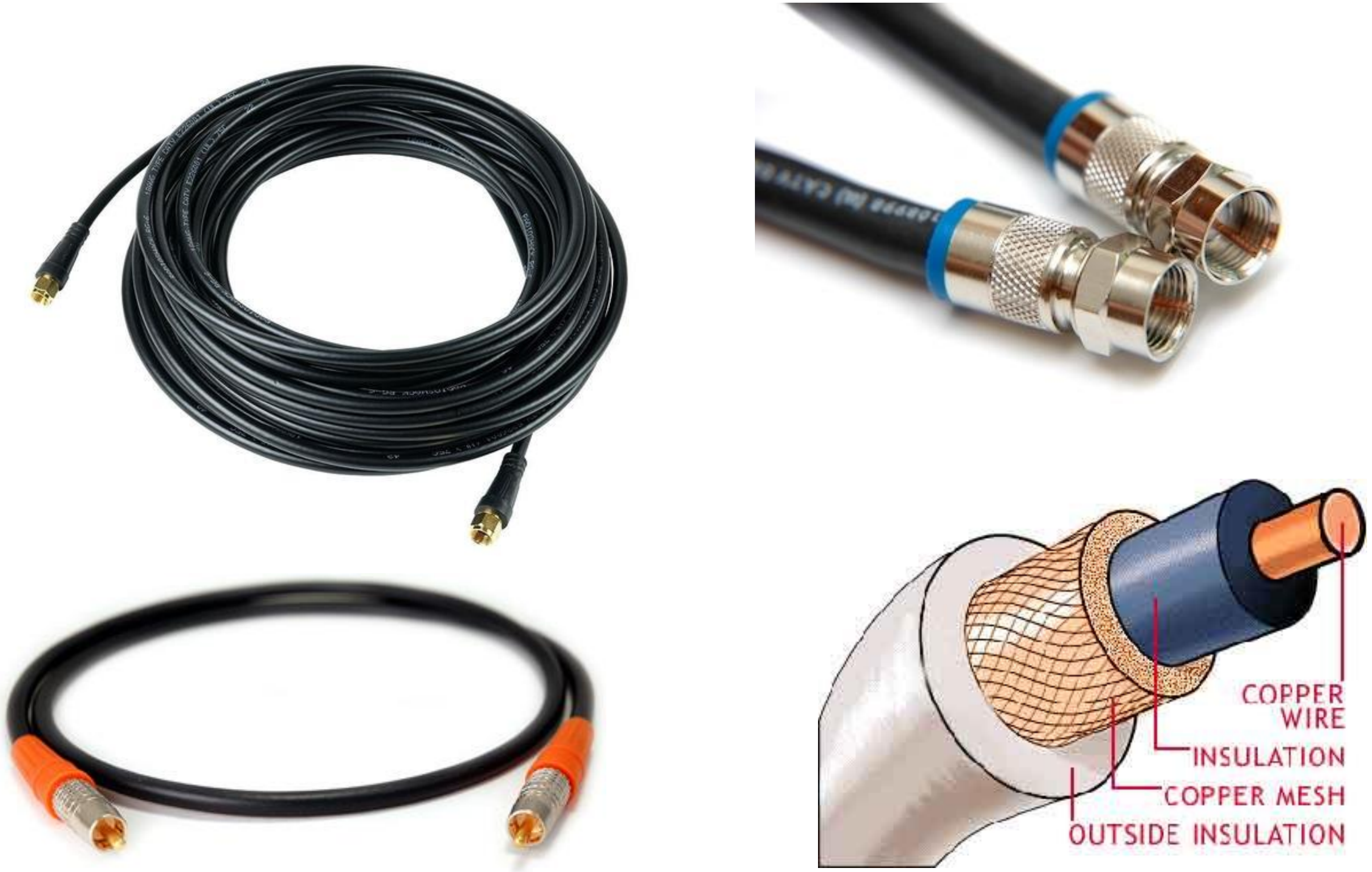


Fig. 3.2 (e) Coaxial Cables.

3.1 The Lumped–Element Circuit Model for a TL (Continued)

3.1.1 Transmission line Definition

- A sinusoidal low frequency voltage is dropped across the resistor. If the supply and resistor is connected by an ideal (negligible length) conductor, it will be in same phase as in Fig. 3.3(b).
- When a sinusoidal high frequency voltage and a quarter wavelength is added between the supply and the resistor, the voltage at the resistor is 110^o out of phase with the supply voltage as in Fig. 3.3(c).

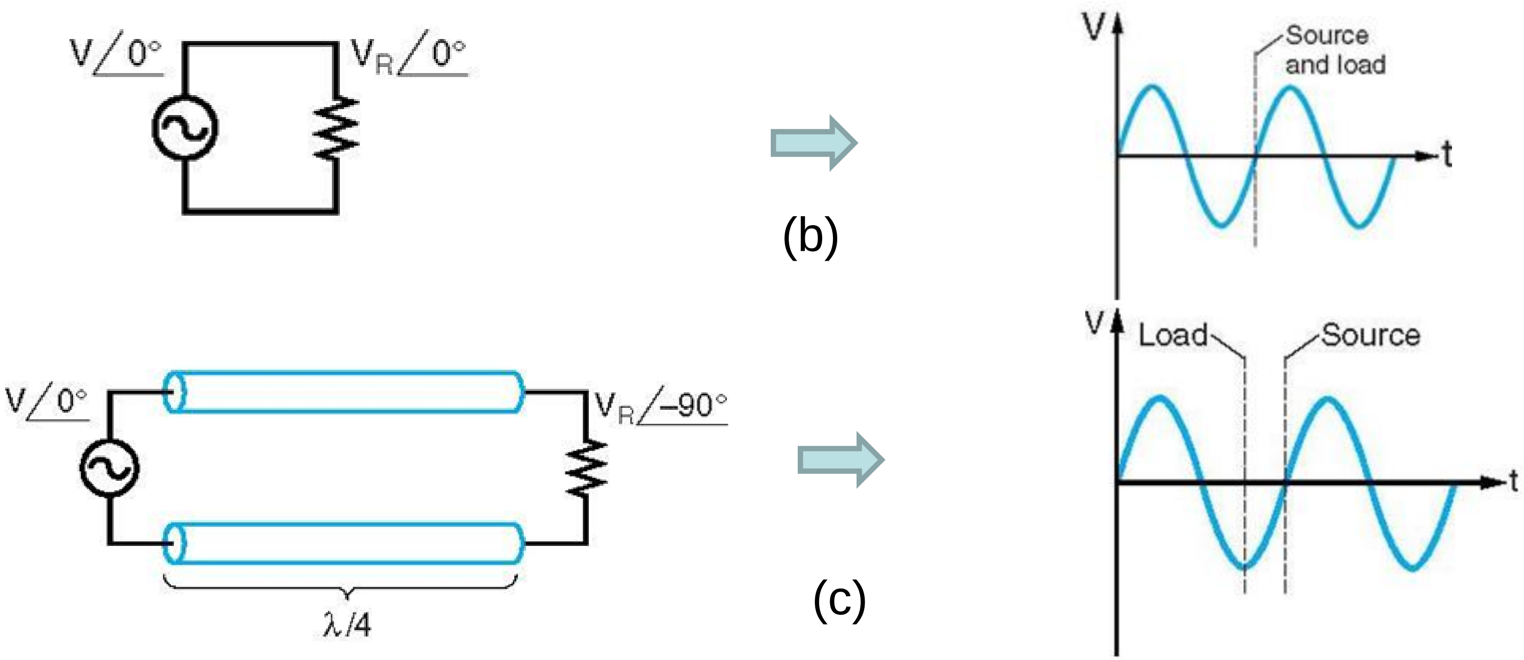


Fig.3.3(b) Low frequency transmission line; (c) High frequency transmission line.

- **In circuit theory** the electrical wavelength is much larger than the physical dimension of the circuits[50hz has wave length =6*10^6m],

- while in microwave frequencies **the transmission line theory** considers the transmission line has a length of many wave lengths (of the wave travelling on it)[10GHz has 3cm wave length].

3.1 The Lumped–Element Circuit Model for a TL (Continued)

3.1.2 Transmission Line Circuit Model

- A transmission line is often schematically represented as a two wire line or conductors as shown in Fig. 3.4 (a).
- The electric and magnetic fields are perpendicular to each other and to the direction of wave propagation, so TEM mode waves.

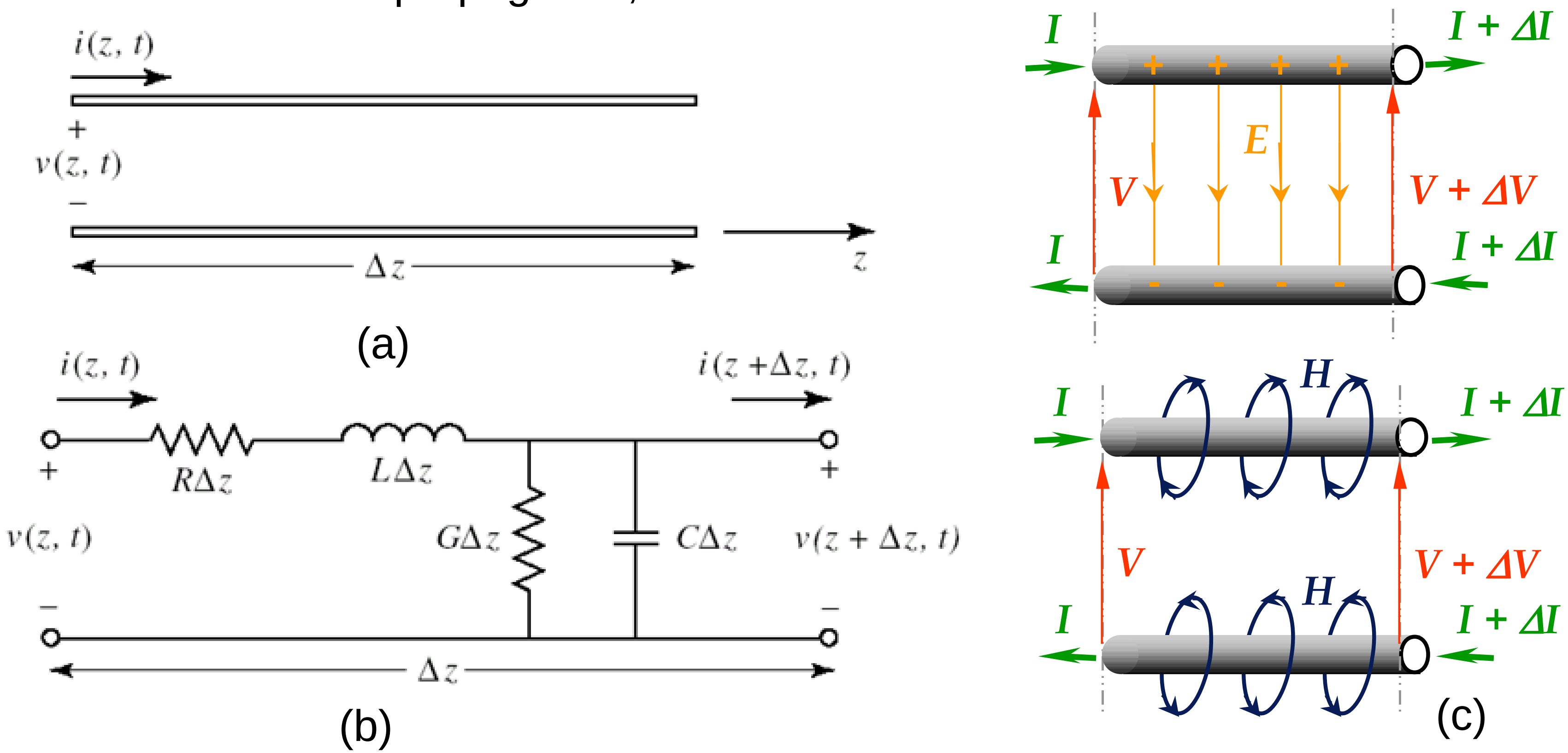


Fig.3.4 transmission representation for an incremental length: (a) Voltage and current definition; (b) Lumped-element equivalent circuit; (c) E and H Fields.

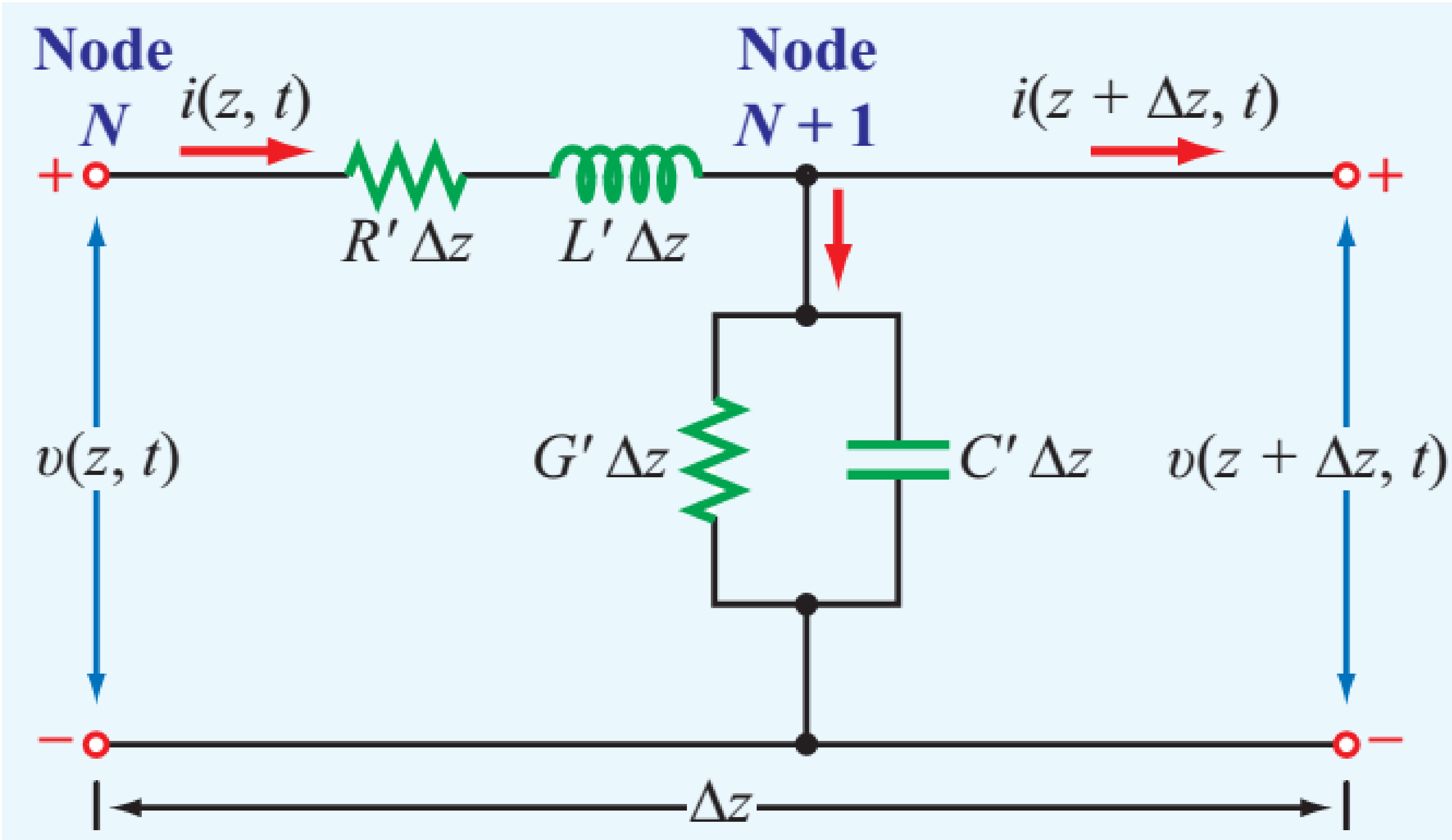
3.1 The Lumped–Element Circuit Model for a TL (Continued)

3.1.2 Transmission Line Circuit Model (Continued)

- We assume propagation in the $\hat{a}_z$ direction. Our model consists of a line section length Δz containing resistance $R\Delta z$, inductance $L\Delta z$, conductance $G\Delta z$ and capacitance $C\Delta z$ as shown in Fig. 3.4(b), where R, L, G and C are per unit length quantities defined as follows:

  R = series resistance per unit length, for both conductors, in Ω/m.
  L = series inductance per unit length, for both conductors, in H/m.
  G = shunt conductance per unite length, in S/m.
  C = shunt capacitance per unit length, in F/m.

- The series inductance L represents the total self-inductance of the two conductors and the shunt capacitance C is due to the close proximity of the two conductors.
- The series resistance R represents the resistance due to the finite conductivity of the conductors, and the shunt conductance G is due to dielectric loss in material between the conductors. R and G, therefore represent Loss.
- A finite length of transmission line can be viewed as a cascade sections of the form shown in Fig. 3.4(b).



$$i(z,t) \neq i(z+\Delta z,t)$$

$$v(z,t) \neq v(z+\Delta z,t)$$

R: resistance /unit length
L: inductance/unit length
C: capacitance/unit length
G:conductance/unit length
So resistance for wire length Δz is R. Δz

3.2 The Transmission Line Equations

3.2.1 Wave Equations of a Transmission Line (Continued)

- From the circuit of Fig. 3.4(b), Kirchhoff's voltage law can be applied to give:

$$-v(z,t)+R\,\Delta z\,i(z,t)+L\,\Delta z\,\frac{\partial i(z,t)}{\partial t}+v(z+\Delta z,t)=0$$

or

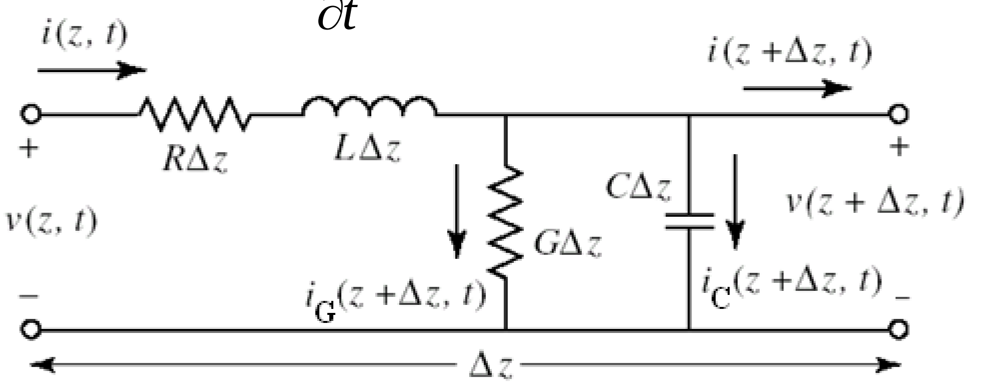
$$v(z+\Delta z,t)-v(z,t)=-R\,\Delta z\,i(z,t)-L\,\Delta z\,\frac{\partial i(z,t)}{\partial t} \qquad (3.1)$$

- The Kirchhoff's current law leads to:

$$i(z,t)=G\,\Delta z\,v(z+\Delta z,t)+C\,\Delta z\,\frac{\partial v(z+\Delta z,t)}{\partial t}+i(z+\Delta z,t)=0$$

$$i(z+\Delta z,t)-i(z,t)=-G\,\Delta z\,v(z+\Delta z,t)-C\,\Delta z\,\frac{\partial v(z+\Delta z,t)}{\partial t} \qquad (3.2)$$

- Dividing (3.1) and (3.2) by Δz and taking the limit as Δz → 0, gives the following differential equations:



$$Lim_{\Delta z\to 0}\frac{v(z+\Delta z,t)-v(z,t)}{\Delta z}$$

$$=-R\;i(z,t)-L\frac{\partial i(z,t)}{\partial t}$$

$$\frac{\partial v(z,t)}{\partial z}=-R\;i(z,t)-L\frac{\partial i(z,t)}{\partial t} \qquad (3.3)$$

Similarly.
$$\frac{\partial i(z,t)}{\partial z}=-G\;v(z,t)-C\frac{\partial v(z,t)}{\partial t} \qquad (3.4)$$

3.2 The Transmission Line Equations (Continued)

3.2.1 Wave Equations of a Transmission Line (Continued)

- These two equations are the time domain form of the transmission line equations, or Telegrapher 's equations. For the sinusoidal steady-state condition, with cosine-based phasor (3.3) and (3.4) simplify to:

$$\frac{\partial V_s(z)}{\partial z} = -(R + j\omega L)I_s(z) \quad (3.5) \qquad \frac{\partial I_s(z)}{\partial z} = -(G + j\omega C)V_s(z) \quad (3.6)$$

- Note the similarity in the form of (3.5) and (3.6) with the Maxwell's curl equations of (3.10) and (3.11).
- The two equations (3.5) and (3.6) can be solved simultaneously or differentiate one with respect to z and substitute in the other, to give the wave equations for V (z) and I(z) as:

$$\frac{\partial^2 V_s(z)}{\partial z^2} - \gamma^2 V_s(z) = 0 \qquad (3.7)$$

$$\frac{\partial^2 I_s(z)}{\partial z^2} - \gamma^2 I_s(z) = 0 \qquad (3.8)$$

- Where γ is the complex propagation constant, which is a function of frequency and is given by:

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)} \qquad (3.9)$$

where α is the attenuation constant and β is the phase constant.

3.2 The Transmission Line Equations (Continued)

3.2.2 Wave Propagation on a Transmission Line (Continued)

- Applying the same approach for solving the uniform plane wave in chapter 2, the traveling wave solution can be found as:

$$V_s(z) = V_o^+ e^{-\gamma z} + V_o^- e^{\gamma z} \qquad (3.10)$$

$$I_s(z) = I_o^+ e^{-\gamma z} + I_o^- e^{\gamma z} \qquad (3.11)$$

- Where $e^{-\gamma z}$ term represent wave propagation in the (+ve) z direction and $e^{\gamma z}$ term represent wave propagation in the (-ve) z direction.
- From (3.10) substitute the voltage V(z) in (3.5), the current on the line is:

$$\frac{\partial V_s(z)}{\partial z} = -(R + j\omega L)\, I_s(z) \Longrightarrow I_s(z) = \frac{\gamma}{R + j\omega L}\left(V_o^+ e^{-\gamma z} - V_o^- e^{\gamma z}\right) \qquad (3.12)$$

- From (3.12) the characteristic impedance, Z_o that relates the voltage and current on the line, can be defined as:

$$\left.\frac{V(z)}{I(z)}\right|_{Forward} = Z_o = -\left.\frac{V(z)}{I(z)}\right|_{Backward} \Longrightarrow Z_o = \frac{R + j\omega L}{\gamma} = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = |Z_o|\, e^{j\theta} \qquad (3.13)$$

- The current on the line has the form:

$$I_s(z) = \frac{V_o^+}{Z_o} e^{-\gamma z} - \frac{V_o^-}{Z_o} e^{\gamma z} \qquad (3.14)$$

3.2 The Transmission Line Equations (Continued)

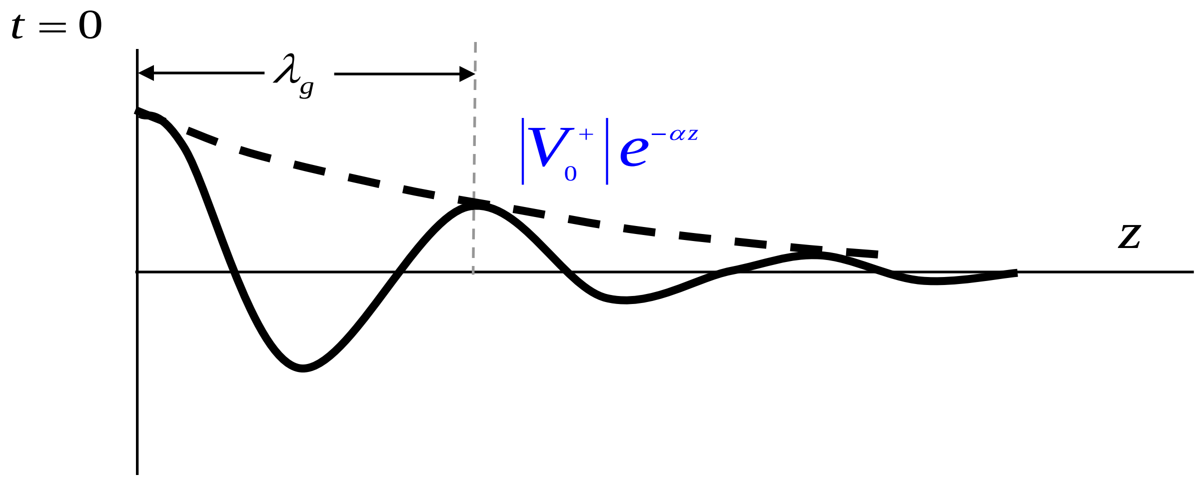
3.2.2 Wave Propagation on a Transmission Line (Continued)

- In time domain, the voltage wave form can be expressed as:

$$v(z,t) = \left|V_o^+\right| e^{-\alpha z}\cos(\omega t - \beta z) + \left|V_o^-\right| e^{\alpha z}\cos(\omega t + \beta z)$$

$$i(z,t) = \left|\frac{V_o^+}{z_o}\right| e^{-\alpha z}\cos(\omega t - \beta z - \theta) - \left|\frac{V_o^-}{z_o}\right| e^{\alpha z}\cos(\omega t + \beta z - \theta) \qquad (3.15)$$

- Where V_0^+ and V_o^- are voltage amplitude, ω is the radian frequency and θ is the characteristic impedance phase angle.



Forward travelling wave (a wave traveling in the positive z direction):

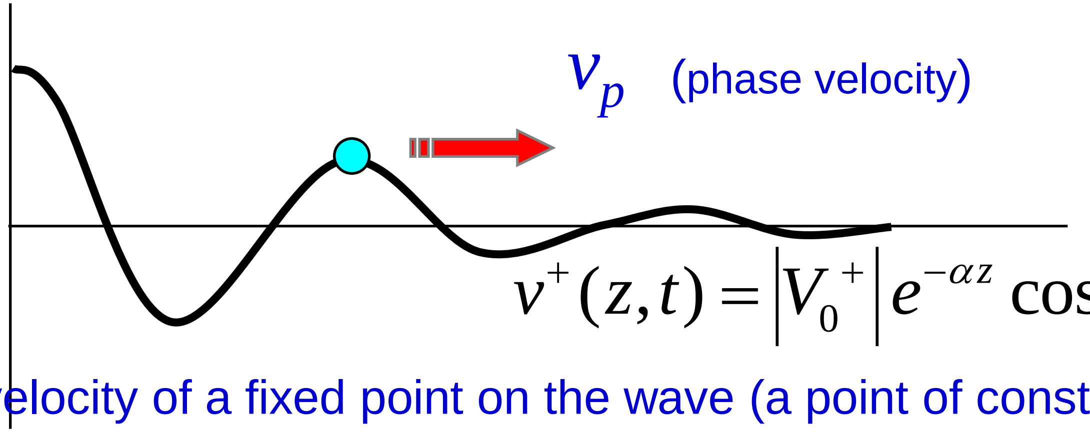
3.2 The Transmission Line Equations (Continued)

3.2.3 Transmission Line wave parameters

- Using the same arguments similar to those of section 2.3.3, we find the wave parameters are:

1. The phase velocity V_p is: $$v_p = \frac{\omega}{\beta} \quad (3.16)$$
2. The wavelength λ is: $$\lambda = \frac{2\pi}{\beta} \quad (3.17)$$
3. The characteristic impedance Z_o is (3.13):

$$\left.\frac{V(z)}{I(z)}\right|_{Forward} = Z_o = -\left.\frac{V(z)}{I(z)}\right|_{Backward} \Rightarrow \quad Z_o = \frac{R + j\omega L}{\gamma} = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = |Z_o| e^{j\theta} \quad (3.13)$$



$$v^+(z,t) = \left|V_0^+\right| e^{-\alpha z} \cos(\omega t - \beta z + \phi)$$

Track the velocity of a fixed point on the wave (a point of constant phase), e.g., the crest.

Thank you for your attention